Products of random matrices and applications in branching processes and branching random walks

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Three lectures are planed to present some recent limit theorems for products of independent and identically distributed random matrices. We focus on precise large deviations and convergence rates in the Gaussian approximation. Applications to multi-type branching processes in random environments and branching random walks will also be presented.

Lecture 1 (1 March 2023): products of random matrices. Let $(g_n)_{n\geq 1}$ be a sequence of independent and identically distributed random matrices of type $d \times d$ with real coefficients. Consider the products $G_n := g_n \dots g_1$ and the random walk $(G_n x)_{n\geq 1}$ in $\mathbb{R}^d$, where $x \in \mathbb{R}^d$ is the starting point. We are interested in asymptotic properties of the operator norm $\|G_n\|$, the vector norm $|G_n x|$, the coefficients $G_n^{i,j}$ (the (i,j)-th entry of G_n), and the spectral radius $\rho(G_n)$ of G_n, as $n \to \infty$. Precise large and moderate deviation results of type Baradur-Rao and Cramér will be presented. Berry-Esseen bound and Edgeworth expansion about the rate of convergence in the central limit theorem will also be considered. (Mainly based on joint works with Ion Grama and Hui Xiao: J. Eur. Math. Soc. 2022, Ann. Prob. 2023, Stochastic Proc. Appl. 2023.)

Lecture 2 (8 March 2023): multitype branching processes in random environments. We consider a branching process with a random environment, in which the reproduction law of generation n depends on the random environment at time n, unlike a constant distribution assumed in the classical Galton-Watson process. The famous Kesten-Stigum theorem on a supercritical multi-type Galton-Watson process gives a precise description of the exponential increasing rate of the population size via a criterion for the non-degeneracy of the fundamental branching martingale. Finding the corresponding result in the random environment case is a longstanding problem. For the single-type case the problem has been solved by Athreya and Kaplin (1971) and Tanny (1988), but for the multi-type case the problem remained open for about 50 years. In this lecture I will explain how to solve this problem, by constructing a suitable martingale which reduces to the fundamental branching martingale in the constat environment case, and by establishing a criterion for the non-degeneracy of its limit. I will then explain how to use this result and limit theorems on products of random matrices to obtain corresponding results on branching processes, such as law of large numbers, central limit theorems, Berry-Essen bound, large and moderate deviation results. (Mainly based on a joint work with Ion Grama and Erwan Pin, Ann. Appl. Prob. 2023.)

Lecture 3 (15 March 2023): branching random walks driven by products of random matrices. A branching random walk is a system of particles, where each particle gives birth to new particles of the next generation, which move on $\mathbb{R}^d$ according to some probability law. In this talk, we consider the new model in which the position of a particle is obtained by the action of a matrix on the position of its parent, where the matrices corresponding to different particles are independent and identically distributed. This permits us to extend significantly the domains of applications of the classical theory of branching random walks where particles move according to an additive random walk. We study the asymptotic properties of the counting measure which counts the number of particles of generation n situated in a given region. A central limit theorem and precise large deviation result will be presented. As an important ingredient of the proof, a sufficient and necessary condition for the non-degeneracy of the limit of the fundamental martingale related to the products of random matrices along branches, will also be presented. (Based on a joint work with Thi Thuy Bui and Ion Grama.)

Bio of the speaker. Quansheng Liu is professor of exceptional class at University of South Brittany (Univ. Bretagne-Sud, France). From 2007 to 2013 and from 2017 to 2022, he has been the Director of the Laboratory of Mathematics of University of South Brittany, and Associate Director of the Laboratory of Mathematics of Atlantic Brittany (LMBA, CNRS UMR 6205). He received his B.S. and M.S. degrees in Mathematics from Wuhan University respectively in 1984 and 1987, and his Ph.D. degree in Probability from University Paris 6 in 1993. His research interests include probability theory and image processing. He has published over 100 research articles in high level journals, including J. Eur. Math. Soc., Prob. Th. Rel. Fields, Ann. Prob., Ann. Appl. Prob., Bernoulli, Stoch. Proc. Appl., Ann. Inst. Henri Poincaré, IEEE Trans. Image Processing, SIAM J. Imaging Sciences, and J. Sci. Comput.